STUDIES ON MIXING. XXXV.*

FLOW PATTERN IN A SYSTEM WITH AXIAL MIXER** AND RADIAL BAFFLES

I. FOŘT, Z. GRAČKOVÁ and V. KOZA

Department of Chemical Engineering,
Institute of Chemical Technology, Prague 6

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The convective flow is described of a liquid driven by a paddle mixer with inclined flat blades in a cylindrical vessel equipped with four radial baffles. The flow pattern is given as a streamline field computed by numerical solution of the Laplace equation. The boundary conditions are those of the first kind (the Dirichlet problem). The model proposed is verified by direct computation of the streamline field determined from the experimentally found velocity field in the system examined. The average relative deviation of the streamline field constructed on the basis of experiments from that following from the solution is better than 20%. It has been established that the flow pattern in a system with axial mixer and radial baffles under the turbulent regime of the charge is affected primarily by the relative size of the mixer and the vessel.

In addition to the mixing due to the turbulent diffusion, the mixer causes also convective flow of liquid ensuring a uniform mixing in the whole volume of the charge. A quantitative picture of the convective flow is the flow pattern. The latter consists of streamlines computed from the time-averaged velocity field. Such form of the streamline field in a mixed system may become a basis for selecting a suitable geometrical arrangement of a batchwise or a continuous reactor, or, eventually, for problems of jacket-to-batch heat transfer, or solid phase suspension in a mixed liquid.

The streamline field at mixing of newtonian liquids by high-speed mixers in a cylindrical vessel with baffles under the turbulent regime of the batch was investigated qualitatively by Rushton and Oldshue[1]. The figures presented by authors are given with minor alterations in the following papers[2,3]. A qualitative determination of the flow pattern for a radial type of mixer (turbine impeller or paddle mixer with vertical blades) in a vessel with or without baffles is given in papers of Nagata and coworkers[4,5]. The streamline field presented here is constructed from the velocity field obtained experimentally using a Pitot tube with several openings, or the Pitot–Prandtl tube. The streamlines are determined graphically from the distribution of the vector of local mean velocity. The details of this method may be found in many monographs, *e.g.*[6]. In the paper of Souza and Pike[7] the flow pattern is drawn in the form of contours of constant stream function — the streamlines — on the basis of analytical solution of the Reynolds equation for the system in question. Apart from the already cited paper[1] the axial type of mixers, the turbine mixer, or the paddle mixer with inclined blades, are dealt by Porcelli and Marr[8] who followed the circulation of a particle in the batch. The authors report the flow pattern with two types of circulation loops: circu-

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lations originating and terminating in the region of the mixer, characterizing the so called primary flow, and circulations originating and terminating outside the region of the mixer, the so called secondary (or induced) flow. On the basis of this model the authors quantitatively describe the convective flow in terms of the pumping capacity of the mixer and induced flow rate. The flow pattern considered contains two so called dead regions where the flow rate is markedly smaller than in other regions of the system. The first of these regions is located around the axis of the vessel, the other, in the form of a spheroid, is surrounded by the streamlines. The studies of the flow pattern in different subregions of a mixed system with axial mixer in a vessel with baffles, obtained partly from experimental investigation of the velocity fields by means of the three opening directional Pitot tube, partly from the results of investigation of radial distribution of axial pressure over the bottom, have been published in earlier papers of this series[9–14]. Three conclusions follow from the results: *1.* The maximum volumetric flow rate in vertical direction (up or down) exists in the plane of the mixer and decreases toward the bottom and the free surface. *2.* The axial (under the rotating mixer also tangential) component of the mean velocity vector prevails in the proximity of the mixer. The radial component appears important near the bottom where the direction of the flow changes from a descending to an ascending one. *3.* Below and above the plane of the mixer, in between of both mentioned streams, there is a region of relatively low flow rates (the dead region), delimited by a hollow cylinder with rounded bases. 4. The radial velocity profile manifests itself markedly in the stream of liquid ejected by the rotating blades of the mixer and in the stream of liquid rising toward the level along the wall of the vessel. In the stream entering the rotating mixer this profile is practically piston-like. These quantitative conclusions contradict the finding of Wolf and Manning[1] that the volumetric flow directed below the plane of the rotating mixer is not a monotone function of the vertical distance from the plane of the mixer, but it rather exhibits a maximum in a certain distance and only then decreases. This conclusion is at odds not only with the above cited papers but also with the conclusions following from the results of determination of the average time of circulation in the system studied[16].

The conclusions following from the papers published about flow pattern in a cylindrical vessel with axial mixer and radial baffles under the turbulent regime of flow of the charge are all consistent except one. However, there is no method proposed thus far for quantitative description of the flow pattern. An attempt is made in this work to determine theoretically the streamline field in a given system with the use of the solution of the Laplace equation and to compare the obtained patterns with the spatial distribution of the streamlines obtained experimentally from the time-averaged velocity field.

THEORETICAL

Let us consider a mixed system (Fig. 1) consisting of a cylindrical vessel with four radial baffles and axial mixer. The system is filled with a newtonian liquid and the clear liquid height, H, equals the inner diameter, D, of the vessel. In this system we delimit two regions: A region under the mixer, V_{I}, limited by the bottom of the vessel, the surface S_{I} and appropriate part of the wall and radial baffles, and, a region above the mixer, V_{II}, limited by the surface S_{II}, the liquid level and appropriate part of the wall and radial baffles. For the system and regions V_{I} and V_{II} we introduce following simplifying assumptions: *1.* The system is closed for mass transfer with surroundings (the boundaries are the bottom, the walls with radial baffles and the liquid level). *2.* The charge is incompressible. *3.* The system is axially symmetric. *4.* The velocity on the liquid level is practically zero. *5.* The flow pattern is related to the steady state. *6.* The flow in regions V_I and V_{II} is irrotational.* Let us define now the following dimensionless variables:

$$R \equiv r/D\,; \qquad Z \equiv z/H\,, \qquad (1a,b)$$

$$W_{ax} \equiv \overline{w}_{ax}/\pi dn\,; \qquad W_{rad} \equiv \overline{w}_{rad}/\pi dn\,, \quad (2a,b)$$

$$\Psi = \psi/nd^3\,. \qquad (3)$$

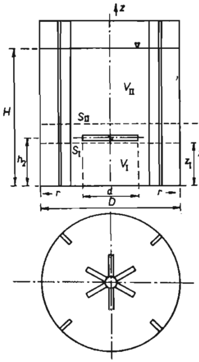

FIG. 1

Sketch of Mixed System and Coordinates

The equation of continuity for the system considered may be written as[17]

$$\frac{\partial(RW_{rad})}{\partial R} + \frac{\partial(W_{ax}R)}{\partial Z} = 0 \qquad (4)$$

and the streamline field in regions V_I and V_{II} may be described by the Laplace equation[17]

$$\frac{\partial^2\Psi}{\partial Z^2} + \frac{\partial^2\Psi}{\partial R^2} - \frac{1}{R}\frac{\partial\Psi}{\partial R} = 0\,. \qquad (5)$$

With reference to the transformation equations $(1a)-(3)$, the stream function for given geometrical arrangement ($H = D$) of the system is defined by the relations

$$W_{rad} = \quad (1/\pi)\,(d/D)^2\,(1/R)\,\partial\Psi/\partial Z\,, \qquad (6a)$$

$$W_{ax} = -(1/\pi)\,(d/D)^2\,(1/R)\,\partial\Psi/\partial R\,. \qquad (6b)$$

Solution of Eq. (5) will be attempted for the boundary conditions given by

$$\Psi \equiv 0\,, \quad Z = Z_p = 0\,; \quad R \in \langle 0;\, 0{\cdot}5\rangle\,, \qquad (7a)$$

* The flow in the cylindrical region between V_I and V_{II} cannot be regarded irrotational owing to the presence of the mixer.

$$\Psi \equiv 0\,, \quad Z = Z_k = 1\,; \qquad R \in \langle 0;\ 0{\cdot}5 \rangle\,, \tag{7b}$$

$$\Psi \equiv 0\,, \quad R = R_k = 0{\cdot}5\,; \quad Z \in \langle 0;\ 1 \rangle\,. \tag{8}$$

The definitions (*7a*)–(*8*) are mathematical expressions of the assumptions 1 and 4. The boundary conditions for the surfaces S_i ($i = \text{I, II}$), through which the liquid enters and leaves the regions V_i ($i = \text{I, II}$), are given by functions

$$\Psi_i = \Psi_i(R)\,, \quad Z_i = \text{const.},\ i = \text{I, II}\,. \tag{9}$$

These functions are obtained from the radial profiles $W_{ax,i} = W_{ax,i}(R)$, ($Z_i = \text{const.}$; $i = \text{I, II}$) found experimentally on the surfaces S_I and S_II, and further from the relations

$$\Psi_i(R) - \Psi_i(R_p) = \pi(D/d)^2 \int_{R_p}^{R} W_{ax,i}(R)\, R\, \mathrm{d}R\,,$$

$$Z_i = \text{const.},\ i = \text{I, II}\,, \tag{10}$$

where the value of the stream function at the point R_p is set equal to zero:

$$\Psi_i(R_p) \equiv 0\,, \quad Z_i = \text{const.},\ \ i = \text{I, II}\,. \tag{11}$$

The coordinate R_p determines that position in an arbitrary profile $W_{ax,j} = W_{ax,j}(R)$, ($Z_j = \text{const.}$), where the axial velocity component reaches zero. That means that, with respect to Eq. (*7b*), the function Ψ has a minimum at $R = R_p$. Eq. (*10*) expresses the radial profile of the stream function Ψ. Similarly we may write for the axial profile of this function*

$$\Psi_j(Z) - \Psi_j(Z_p) = \pi(D/d)^2\, R \int_{Z_p}^{Z} W_{rad,j}(Z)\, \mathrm{d}Z\,,$$

$$R_j \in \langle R_p;\ R_k \rangle\,; \quad j = 1, 2, 3, \ldots\,, \tag{12}$$

where with respect to the definitions (*7a*) and (*7b*) we have

$$\Psi_j(Z_p) \equiv \Psi_j(Z_k) = 0, \quad R_j \in \langle R_p;\ R_r \rangle; \quad j = 1, 2, 3, \ldots\,. \tag{13}$$

By means of the relations (*10*) and (*12*) and with the aid of Eqs (*11*) and (*13*) we can calculate numerically both the axial and the radial profiles of the stream function. From the knowledge of the distribution of the stream function Ψ along the bounda-

* With respect to the formulation of the problem, Eqs (*10*) and (*12*) hold in V_I, V_II and on their boundaries.

ries of V_I and V_{II} we can find the field of streamlines by a numerical solution of the partial differential equation (5). This provides a set of coordinates assigned to an arbitrary value of the stream function satisfying the boundary conditions (7a)−(9). The problem posed by the solution is the boundary value problem of the first kind, or the Dirichlet problem, which can be solved for the system considered by some standard method on a computer[6]. The flow pattern in the whole mixed system, including the cylindrical part between V_I and V_{II}, may be obtained by graphical or numerical interpolation of the function $\Psi = \Psi(R, Z)$ over the region in question.

EXPERIMENTAL

The determination of the velocity field. The field of velocities in the mixed system was evaluated from the results of pressure measurements with the directional Pitot tubes[10,12,13]. The experiments were carried out in a cylindrical perspex vessel with flat bottom, $D = 290$ mm in diameter, filled with distilled water at 20°C. The height of clear liquid, H, was equal to the diameter D. The vessel was provided with four radial baffles, $0{\cdot}1D$ wide. A six-paddle mixer with flat blades inclined at an angle of 45° was used[16,18]. The mixer was located in the axis of the vessel and rotated always in such a direction so as to drive the liquid toward the bottom.*

Three relative dimensions $d/D = 1/3, 1/4, /15$ of mixers were used in experiments. The velocity measurements in the charge were carried out at three frequencies of revolution for each mixer. The relative height of the mixer above the bottom was $h_2/D = 1/4$ for all mixers used. The measurement of the pressure distribution with the directional probes was carried out in two ways: *1.* Near the bottom and the walls of the vessel, the pressure was measured by means of hypodermic needles connected to the manometers and adapted into the shape of three types of directional probes[13]. *2.* At a greater distance from the bottom and the walls, the pressure was measured by means of the three opening Pitot tube[10]. Thus the courses were obtained of the pressure detected by the directional probes in several directions in different axial (vertical) distances, Z_j, above the bottom (Table I). From these pressures the quantities $\bar{w}$, $\bar{w}_{ax}$, $\bar{w}_{rad}$, or their dimensionless equivalents $\mathbf{W}$, W_{ax} and W_{rad}, were obtained in a given point r_j (or R_j) on a given ray of the coordinate z_j (or Z_j) by the method described in the cited papers[10,12,13] of this series. Thus the radial profiles $W_{ax,j} = W_{ax,j}(R)$, (Z_j = const.), and $W_{rad,j} = W_{rad,j}(R)$, (Z_j = const.), were obtained and further averaged from results obtained at three different frequencies of revolution of the mixer at a given relative size d/D. For the region near the bottom, where the determination of W_{rad} carries much smaller error than that of W_{ax} owing to the predominance of the radial component over the axial one, the axial profiles $W_{rad,j} = W_{rad,j}(Z)$, (R_j = const.), were obtained from the already known radial profiles of W_{rad} graphically.

The determination of the streamline field. The radial profiles of Ψ were calculated for all selected distances Z_j (Table I) from the velocity profiles $W_{ax,j} = W_{ax,j}(R)$, (Z_j = const.), and with the aid of Eq. (*11*). To calculate the courses of this function, a relation analogous to Eq. (*10*) for different Z_j was used:

* The arrangement of the mixed system as well as the choice of the direction of motion of liquid driven by the mixer were made in accord with the arrangement used in a majority of practical applications *e.g.*: homogenation of miscible liquids, dissolution and suspension of solid phase in a batch or continuous system[1].

$$\Psi_j(R) - \Psi_j(R_p) = \pi(D/d)^2 \int_{R_p}^{R} W_{ax,j}(R)\, R\, dR\,,$$
$$Z_j \in \langle Z_p;\ Z_I \rangle\,, \quad j = 1, 2, 3, \ldots \qquad (14)$$
$$Z_j \in \langle Z_{II};\ Z_k \rangle\,, \quad j = 1, 2, 3, \ldots$$

Thus the radial profiles of the stream function were obtained in several vertical distances Z_j from the bottom (Table I). These profiles were further utilized as boundary conditions for the differential equation (5) (relation (9)) on one hand, and for comparison of the functions $\Psi_j = \Psi_j(R)$, (Z_j = const.), obtained theoretically on the other hand. The profiles $\Psi_j = \Psi_j(Z)$, (R_j = const.), were computed in the proximity of the bottom, *i.e.* in the interval $Z \in \langle 0{\cdot}000;\ 0{\cdot}160 \rangle$

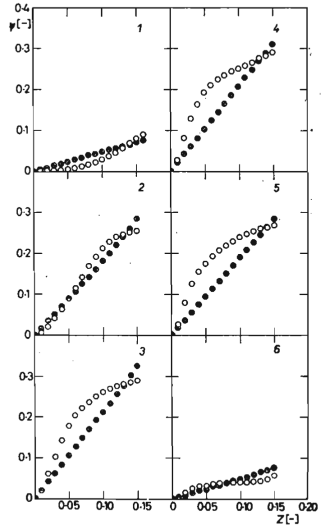

FIG. 2

Axial Profile of the Stream Function (d/D = = 1/5)

○ Calculated from experimental velocity field, ● solution of the Laplace equation for V_I. *1* $R = 0{\cdot}100$; *2* $R = 0{\cdot}175$; *3* $R = 0{\cdot}250$, *4* $R = 0{\cdot}300$; *5* $R = 0{\cdot}350$; *6* $R = 0{\cdot}450$.

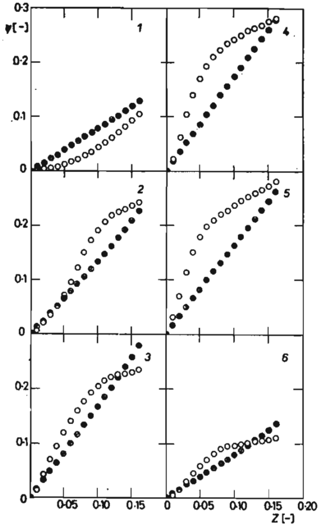

FIG. 3

Axial Profile of the Stream Function (d/D = = 1/4)

○ Calculated from experimental velocity field, ● solution of the Laplace equation for V_I. *1* $R = 0{\cdot}100$; *2* $R = 0{\cdot}175$; *3* $R = 0{\cdot}225$, *4* $R = 0{\cdot}300$; *5* $R = 0{\cdot}350$; *6* $R = 0{\cdot}450$.

from the velocity profiles $W_{rad,j} = W_{rad,j}(Z)$, (R_j = const.), by means of the relation (*12*) and assuming validity of Eq. (*13*). Thus obtained axial profiles of the quantity Ψ were compared with the results of the theoretical solution, *i.e.* the profiles $\Psi_j = \Psi_j(Z)$, (R_j = const.), the obtained by solving the partial differential equation (*5*) with the boundary conditions (*7a*)—(*9*). A numerical solution of the Laplace equation (*5*) with the boundary conditions (*7a*)—(*9*) was carried out

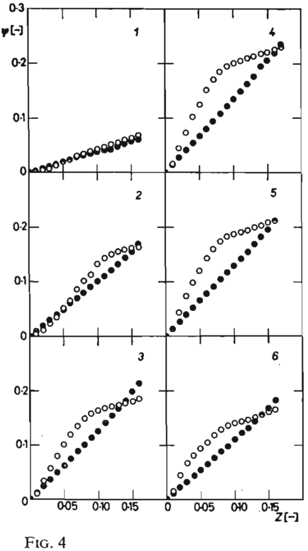

FIG. 4

Axial Profile of the Stream Function (d/D = = 1/3)

○ calculated from experimental velocity field, ● solution of the Laplace equation for V_I. *1* R = 0·150; *2* R = 0·200; *3* R = 0·250, *4* R = 0·300; *5* R = 0·350; *6* R = 0·450.

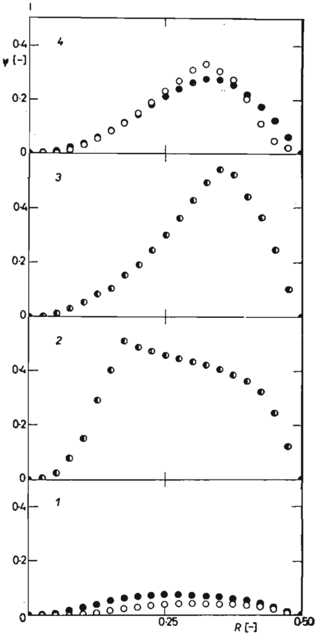

FIG. 5

Radial Profiles of the Stream Function ($d/D = 1/5$)

○ Calculated from experimental velocity field, ● solution of the Laplace equation for V_i (i = I, II) ◑ boundary condition for the Laplace equation for region V_i (i = **I, II**); *1* Z = 0·0345; *2* Z_I = 0·207; *3* Z_{II} = 0·310; *4* Z = 0·380.

using the over-relaxation method[6,19] with optimization of the relaxation factor. After preliminary tests the magnitude of the increments was set equal to $\Delta R = \Delta Z = 0{\cdot}01$. The computation for the given grid and accuracy $\Delta\Psi = 1{\cdot}0 \,.\, 10^{-5}$ (given as the maximum difference of two consecutive iterations) took about one hour on an Elliot 4120 computer for one region V_i (i = I, II). For the type of problem in question the optimum value of the relaxation factor was found equal 1·50. The result of the numerical computation was the set of values of the stream function in the points of the grid enabling by interpolation the coordinates of the streamlines to be determined.

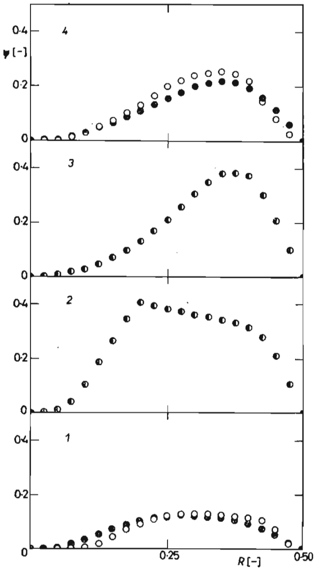

FIG. 6

Radial Profile of the Stream Function (d/D = = 1/4)

○ Calculated from experimental velocity field, ● solution of the Laplace equation for V_i (i = I, II), ◑ boundary condition for the Laplace equation and V_i (i = I, II); *1* Z = = 0·0345; *2* Z_I = 0·207; *3* Z_{II} = 0·310; *4* Z = 0·380.

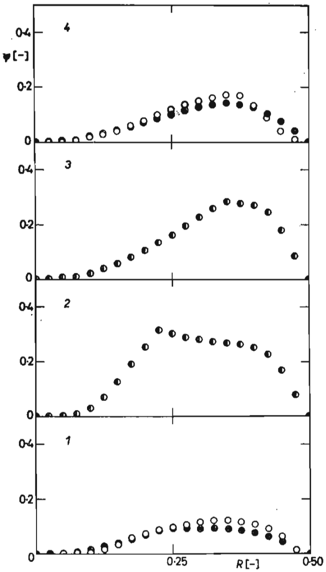

FIG. 7

Radial Profile of the Stream Function (d/D = = 1/3)

○ Calculated from experimental velocity field, ● solution of the Laplace equation for V_i (i = I, II(, ◑ boundary condition for the Laplace equation and V_i (i = I, II); *1* Z = = 0·0345; *2* Z_I = 0·207; *3* Z_{II} = 0·310; *4* Z = 0·380.

TABLE I

Axial Coordinates of the Radial Rays Determining the Points of Measurements with Directional Pitot Tubes

Number of radial ray	1	2	3	4	5	6	7
Z_j	0·0172	0·0345	0·069	0·104	0·207[a]	0·310[b]	0·380

[a] Z_I; [b] Z_{II}.

TABLE II

Average Deviation of the Theoretical Value of $\Psi_j(Z)$, (R_j = const.), obtained from the Laplace Equation from Experimental Ones

d/D	(—)	1/3	1/4	1/5
σ_Ψ	(%)	21·4	22·2	19·5

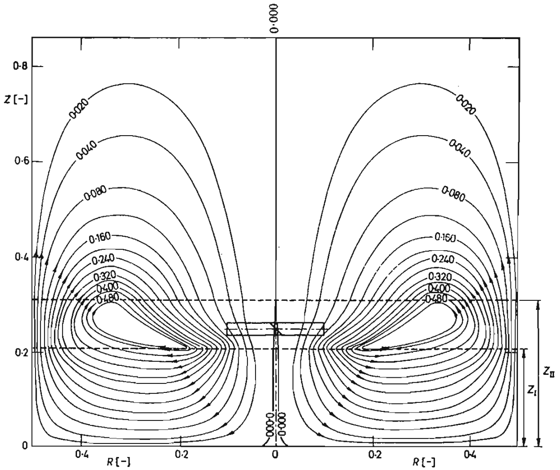

FIG. 8

Field of Streamlines in a Cylindrical System with Axial Mixer and Radial Baffles ($d/D = 1/5$)

RESULTS AND DISCUSSION

Comparison of Experimental and Theoretical Field of Streamlines

Figs 2–4 and Figs 5–7 show the axial respectively the radial profiles of the stream function obtained from experimental measurements of the velocity field and those obtained from the solution of the Laplace equation (relation (5)) for the boundary conditions (7a)–(9). The last boundary condition $\Psi_i = \Psi_i(R)$, (Z_i = const.; i = = I, II) is plotted in Figs 5–7. The course of this function exhibits considerable crowding of the streamlines exiting from the region V_I as well as those entering the region V_{II}. The same phenomenon may be observed in the stream entering the region V_I after passage of the liquid through the space occupied by the rotating mixer (the liquid driven by the blades of the mixer) and was proven also in the region V_I in the radial ray with the axial coordinate smaller than Z_I of the surface[14] S_I. This fact manifests itself further by compression of the streamlines near the bottom where the experimentally found functions $\Psi_j = \Psi_j(Z)$, (R_j = const.), display a steeper slope than those calculated from Eq. (5). The behaviour of the velocity field outside the examined region (*i.e.* the cylindrical space between V_I and V_{II}) cannot be taken into consideration for description of the streamline field by the Laplace equation and the

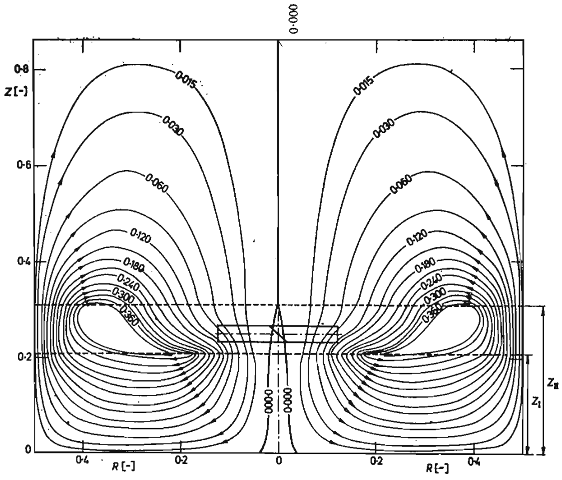

FIG. 9

Field of Streamlines in a Cylindrical System with Axial Mixer and Radial Baffles ($d/D = 1/4$)

knowledge of the behaviour of Ψ on the boundaries of V_I alone (the boundary condition (9)) does not suffice for determining the compression of the streamlines near the bottom expressed by the mentioned steeper slope of the function $\Psi_j = \Psi_j(Z)$, (R_j = const.), in immediate vicinity of the bottom. Accordingly, the systematic deviation decreases with increasing coordinate Z and its values depend both on the geometric arrangement in the mixed system and the coordinate R_j. Table II summarizes the average deviation of the theoretical values of $\Psi_j = \Psi_j(Z)$, (R_j = const.) from those found experimentally near the bottom, *i.e.* in the range $Z \in \langle 0{\cdot}000; 0{\cdot}0160 \rangle$. The average values of the deviations, σ_Ψ, are given in dependence on the relative size of the mixer d/D. From Table II it follows that the flow pattern near the bottom given by the streamlines in this region may be predicted on the basis of the model proposed with the accuracy of about 20%. From comparison of the functions $\Psi_j = \Psi_j(R)$, (Z_j = const.), calculated from experimental results and those from the solution of Eq. (5), it may be concluded that in all six cases compared the differences are insignificant with respect to inaccuracies of determination of the profiles $\Psi_j = \Psi_j(R)$, (Z_j = const.), from corresponding experimental profiles of the axial component of local velocity. However, systematic deviations appear again and their origin is identical with that given above for the differences between the theoretical and experimental course

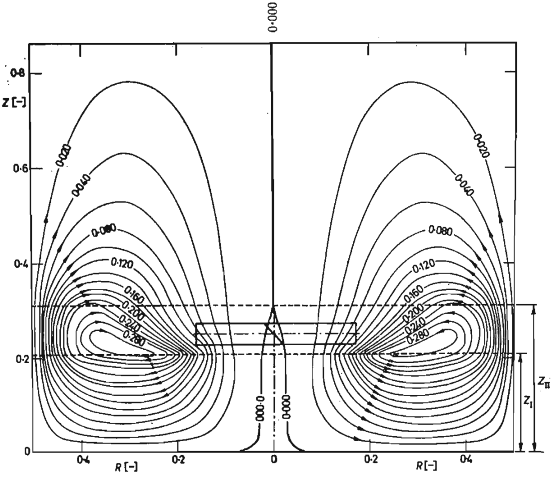

FIG. 10

Field of Streamlines in a Cylindrical System with Axial Mixer and Radial Baffles ($d/D = 1/3$)

of the stream function. The description of the streamline field in a system with axial mixer and radial baffles by the Laplace equation may be regarded as an approximation for region V_I, and above all for V_{II}, which differs from reality within errors mentioned above.

Field of Streamlines in a System with Axial Mixer and Radial Baffles

Figs 8–10 show the field of streamlines in the system examined, obtained from the above described solution of Eq. (*5*). The contours of constant stream function form the flow pattern, since, as it is known[17], the stream function is constant along a streamline. The streamlines passing through the cylinder between the regions V_I and V_{II} were obtained by graphical interpolation of the curves of appropriate values of the stream function and by making use of the conclusions of paper[12] dealing with the flow in this volume. It was not possible to solve the Laplace equation for the volume between the region V_I and V_{II} since it cannot be expected that assumption 6 would hold there[12,20]. It was found on the basis of the preliminary calculations that Eq. (*5*) cannot be solved for the boundary condition (*9*) specified at the inlet and the exit of the region occupied by the rotating mixer either and, consequently, that the solution of the Laplace equation cannot be obtained for the whole system at once. The reason was that the results of this approach differed markedly and systematically from the results of experimental determination of the streamline field and it was therefore necessary to divide the volume of the mixed batch into the volumes V_I and V_{II} and to solve Eq. (*5*) there separately.

Numerical values assigned to individual curves in Figs 8–10 (numbers with four digits) are the appropriate values of the stream function. Broken line show the boundaries of the regions V_I and V_{II}, *i.e.* the surfaces S_I and S_{II}. From the flow patterns found it follows that their shape is affected primarily by the geometric arrangement of the mixed system. The field of streamlines is deformed by the source of the convective flow (the mixer) and the deformation is the greater the smaller is the relative size of the mixer. At the same time the maximum value of the stream function increases in a given cross-section. This dependence was evaluated quantitatively[10–12,16] and the following proportionality for the axial distance from the bottom was obtained

$$\Psi_{\max, Z=\mathrm{const.}} = C(d/D)^{-1}\,, \tag{15}$$

where the parameter C decreases with increasing distance from $Z = h_2/D$. The flow pattern is affected primarily in the region below the rotating mixer (region V_I), since the lower radius of the cone through which the liquid passes from the volume V_{II} into V_I changes markedly[12] with the relative size of the mixer and the vessel (Figs 5–7). This factor, however, affects not only the course of streamlines at downward flow but also their course near the bottom and at upward flow where even a relatively smaller mixer causes a greater compression of the streamlines. The turns of the streamlines below the rotating mixer do not appear the same for all streamlines. Practically all liquid changes its direction by 180°. In doing so one part turns gradually, *i.e.* in two turns approximately by 90°. In between it flows horizontally along the bottom. The other part travels on paths uniformly turning from the original direction into the reverse one. This fact confirms the considerations regarding the flow pattern following from the results of measurement of the force action of the flow on the bottom[11].

The character of the streamlines near the bottom explains also why radial profile of axial pressure on the bottom exhibits a section of subatmospheric pressure. A higher density of streamlines near the bottom points to an increase of kinetic energy in this region (indicated by pressure probes) and consequently the static pressure must decrease in order that the mechanic energy balance is preserved. The flow pattern above the rotating mixer (region V_{II}) is not affected very markedly by the relative size of the mixer since the distribution of the streamlines over the surface S_{II} is independent[12] of the ratio d/D (5–7).

The flow pattern obtained corresponds better with the qualitative considerations published earlier[1,8]. The deformation of the streamlines under the mixer has not been published to date and could not be taken into consideration in qualitative construction of flow patterns. The intensity of convective flow in a mixed charge, defined as a flow per unit area (or in the proposed construction of the flow pattern as a number of streamlines per unit area), in different parts of the system can be compared with the results of measurement of spatial homogeneity of the charge[21]. The regions insufficiently affected by the convective flow need a substantially longer time to achieve a requested homogeneity than those with intensive convection. Although the rate of homogenation is affected also by turbulent diffusion, the importance of uniform intensive flow in the whole charge stands out from the results of such measurements when the former form of mass transfer diminishes owing to the increasing viscosity. When the effect of turbulent diffusion is diminished the field of contours of constant rate of homogenation approximates the field of streamlines: the contours of constant stream function. The region of insufficient mixing is then almost identical with the "dead region" passed, at the given grid, by no streamline[21]. Another "dead region", located under the mixer along the axis of the vessel, is interesting more from the point of view of solid suspension as a region where settling of the solids occurs and the sediments form[2]. The flow pattern obtained confirms also the justification of the wall-to-batch heat transfer model[22]. This model is based on assumption of different character of flow near the wall and bottom and hereby on different mechanism of wall heat transfer in these regions: Near the bottom the character of the flow is dissipative, induced by the change of direction of flow. Near the wall the flow is practically without dissipation because the flow is straight and no sudden turns occur. These facts (in cit.[22] proven by heat transfer measurements in both regions) are confirmed by the flow pattern obtained and moreover the existence of dissipative region near the bottom is indicated by the results of spatial inhomogeneity of the rate of dissipation of mechanical energy in the system examined[12].

The results may be useful also for description of the streamline field in a homogeneous continuous stirred reactor as long as the ratio of the volumetric flow rate to the pumping capacity of the mixer is negligible. Even in those cases, however, when this stipulation is not met, conclusions can be drawn for design purposes, such as *e.g.*: location of the inlet, outlet *etc.*, ensuring that the mixing effect of the stirrer is made best use of. That means that the substance added is brought by the convective flow into the proximity of the rotating mixer before withdrawing the product. Location of the inlet immediately above the mixer and the outlet into the space above the mixer near the wall (preferably in several places distributed evenly on the circumference of the vessel) will ensure that fresh reaction mixture will be homogenized rapidly in region V_I.

LIST OF SYMBOLS

C constant in Eq. (*15*)
D diameter of vessel (m)
d diameter of mixer (m)
H clear liquid height (m)
h_2 height of center of mixer above bottom (m)
i summation index
j summation index
k index denoting final value for variables R and Z
n frequency of revolution of mixer (s^{-1})
p index denoting initial value for variables R and Z
R dimensionless radial coordinate
r radial coordinate
S_I surface separating V_I from the rest of charge (m^2)
S_{II} surface separating V_{II} from the rest of charge (m^2)
V_I volume of region under mixer (m^3)
V_{II} volume of region above mixer (m^3)
$\overline{\mathbf{W}}$ dimensionless mean velocity vector
W_{ax} axial component of vector $\overline{\mathbf{W}}$
W_{rad} radial component of vector $\overline{\mathbf{W}}$
$\mathbf{W}$ absolute value of vector $\overline{\mathbf{W}}$
$\overline{w}$ absolute value of mean velocity vector $\overline{\mathbf{w}}$ ($m\,s^{-1}$)
$\overline{w}_{ax}$ axial component of vector $\overline{\mathbf{w}}$ ($m\,s^{-1}$)
$\overline{w}_{rad}$ radial component of vector $\overline{\mathbf{w}}$ ($m\,s^{-1}$)
Z dimensionless coordinate of axial distance
z coordinate of axial distance (m)
ψ stream function ($m^3\,s^{-1}$)
Ψ dimensionless stream function
σ_Ψ average deviation of theoretical and experimental values of function $\Psi_j(Z)$, (R_j = const.)

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